Fill Ups of Probability, Past year Questions JEE Advance

Q.1. For a biased die the probabilities for the different faces to turn up are given below :

Face	1	2	3	4	5	6
Prob.	0.1	0.32	0.21	0.15	0.05	0.17

This die is tossed and you are told that either face 1 or face 2 has turned up. Then the probability that it is face 1 is................... (1981 - 2 Marks)

Ans. Sol. Let E_1 ≡ face 1 has turned up, E2 ≡ face 1 or 2 has turned up.

By the given data $P(E_2) = 0.1 + 0.32 = 0.42$, $P(E_1 \cap E_2) = P(E_1) = 0.1$

Given that E_2 has happened and we have to find the probability of happening of E_1.

∴ By conditional probability theorem, we have

$$P(E_1/E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{0.1}{0.42} = \frac{10}{42} = \frac{5}{21}.$$

Q.2. P(A ∪ B) = P(A∩B) if and only if the relation between P(A) and P(B) is .... (1985 - 2 Marks)

Ans. Sol. Given that $P(A \cup B) = P(A \cap B) \Rightarrow P(A) + P(B) - P(A \cap B) = P(A \cap B)$

$\Rightarrow [P(A) - P(A \cap B)] + [P(B) - P(A \cap B)] = 0$

But $P(A) - P(A \cap B)$, $P(B) - P(A \cap B) \geq 0$ [Q $P(A \cap B) \leq P(A), P(B)$]

$\Rightarrow P(A) - P(A \cap B) = 0$ and $P(B) - P(A \cap B) = 0$ [Q Sum of two non-negative no's can be zero only when these no's are zeros]

$\Rightarrow P(A) = P(B) = P(A \cap B)$

Q.3. A box contains 100 tickets numbered 1, 2, ..., 100. Two tickets are chosen at random. It is given that the maximum number on the two chosen tickets is not more than 10. The minimum number on them is 5 with probability ....

(1985 - 2 Marks)

Ans. Sol. Let A be the event that max. number on the two chosen tickets in not more than 10, and B is the event that min. number on them is 5. We have to find P (B / A).

We know that $P(B/A)=\frac{P(B\cap A)}{P(A)}$

Total ways to select two tickets out of 100 = ${}^{100}C_2$.

Number of ways favourable to A = number of ways of selecting any 2 numbers from 1 to 10 = ${}^{10}C_2 = 45$

A ∩ B contains one number 5 and other greater than 5 and ≤ 10

So ways favourable to A ∩ B = ${}^{5}C_1 = 5$ Therefore, $P(A)=\frac{45}{{}^{100}C_2}$ and $P(B\cap A)=\frac{5}{{}^{100}C_2}$

Thus, $P(B/A)=\frac{5/{}^{100}C_2}{45/{}^{100}C_2}=\frac{5}{45}=\frac{1}{9}$

Q.4. If $\frac{1+3p}{3}, \frac{1-p}{4}, \frac{1-2p}{2}$ are the probabilities of three mutually exclusive events, then the set of all values of p is .... (1986 - 2 Marks)

Ans. Sol. Let $P(A)=\frac{1+3p}{3}$, $P(B)=\frac{1-p}{4}$, $P(C)=\frac{1-2p}{2}$

As A, B and C are three mutually exclusive events

$\therefore$ P (A) + P (B) + P (C) ≤ 1

$\Rightarrow \frac{1+3p}{3}+\frac{1-p}{4}+\frac{1-2p}{2}\le 1$

$\Rightarrow 4 + 12p + 3 - 3p + 6 - 12p \le 12$

$\Rightarrow 3p \ge 1 \Rightarrow p \ge 1/3$ … (i)

Also $0\le P(A)\le 1 \Rightarrow 0\le \frac{1+3p}{3}\le 1$

$\Rightarrow 0 \le 1 + 3p \le 3$

$\Rightarrow -\frac{1}{3} \le p \le \frac{2}{3}$ … (ii)

$0 \le P(B) \le 1 \Rightarrow 0 \le \frac{1-p}{4} \le 1$

$\Rightarrow 0 \le 1 - p \le 4$

$\Rightarrow -3 \le p \le 1$ … (iii)

$0 \le P(C) \le 1 \Rightarrow 0 \le \frac{1-2p}{2} \le 1 \Rightarrow -\frac{1}{2} \le p \le \frac{1}{2}$ … (iv)

Combining (i), (ii), (iii) and (iv), we get $\frac{1}{3} \le p \le \frac{1}{2}$

Q.5. Urn A contains 6 red and 4 black balls and urn B contains 4 red and 6 black balls. One ball is drawn at random from urn A and placed in urn B. Then one ball is drawn at random from urn B and placed in urn A. If one ball is now drawn at random from urn A, the probability that it is found to be red is .... (1988 - 2 Marks)

Ans. Sol. There may be following cases: **Case I :** Red from A to B and red from B to A then prob. of

drawing a red ball from $A = \frac{6}{10} \times \frac{5}{11} \times \frac{6}{10} = \frac{180}{1100} = \frac{18}{110}$

Case II : Red from A to B and black from B to A then prob. of drawing a red from $A = \frac{6}{10} \times \frac{6}{11} \times \frac{5}{10} = \frac{180}{1100} = \frac{18}{110}$

Case III : Black from A to B and red from B to A then prob. of drawing red

from $A = \frac{4}{10} \times \frac{4}{11} \times \frac{7}{10} = \frac{56}{550}$

Case IV : Black from A to B and black from B to A then prob. of drawing red from $A = \frac{4}{10} \times \frac{7}{11} \times \frac{6}{10} = \frac{168}{1100} = \frac{84}{550}$

∴ The required prob $= \frac{18}{110} + \frac{18}{110} + \frac{56}{550} + \frac{84}{550}$

$= \frac{90+90+56+84}{550} = \frac{320}{550} = \frac{32}{55}$

Q.6. A pair of fair dice is rolled together till a sum of either 5 or 7 is obtained. Then the probability that 5 comes before 7 is .... (1989 - 2 Marks)

Ans. Sol. Probability of getting a sum of $5 = \frac{4}{36} = \frac{1}{9}$ = P(A) as favourable cases are {(1, 4), (4, 1), (2, 3), (3, 2) } Similarly favourable cases of getting a sum of 7 are {(1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3)} = 6

∴ Prob. of getting a sum of $7 = \frac{6}{36} = \frac{1}{6}$

∴ Prob. of getting a sum of 5 or 7

$= \frac{1}{6} + \frac{1}{9} = \frac{5}{18}$ [as events are mutually exclusive.]

∴ Prob of getting neither a sum of 5 nor of 7 $= \frac{1}{6} - \frac{5}{18} = \frac{13}{18}$

Now we get a sum of 5 before a sum of 7 if either we get a sum of 5 in first chance or we get neither a sum of 5 nor of 7 in first chance and a sum of 5 in second chance and so on.

Therefore the required prob. Is

$= \frac{1}{9} + \frac{13}{18} \times \frac{1}{9} + \frac{13}{18} \times \frac{13}{18} \times \frac{1}{9} + \ldots \infty = \frac{1/9}{1-13/18} = \frac{1}{9} \times \frac{18}{5} = \frac{2}{5}$

Q.7. Let A and B be two events such that P (A) = 0.3 and P (A ∪ B) = 0.8. If A and B are independent events then P (B) = ... (1990 - 2 Marks)

Ans. Sol. P (A ∪ B) = 0.8

⇒ P (A ∪ B) = P (A) + P (B) – P (A ∩ B)

⇒ P (A ∪ B) = P (A) + P (B) – P (A) P (B)

[As A and B are independent events]

⇒ 0.8 = 0.3 + P (B) – 0.3 P (B)

⇒ 0.5 = 0.7 P (B) ⇒ P (B) = 5/7

Q.8. If the mean and the variance of a binomial variate X are 2 and 1 respectively, then the probability that X takes a value greater than one is equal to .... (1991 - 2 Marks)

Ans. Sol. For a binomial distribution, we know, mean = np and variance = npq

∴ np = 2; npq = 1 ⇒ q = 1/2 ⇒ p = 1/2 and n = 4

∴ P (X > 1) = P (X = 2) + P (X = 3) + P (X = 4)

= 1– P (X = 0) – P (X = 1)

$$=1-{}^4C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^4-{}^4C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^3=1-\frac{1}{16}-\frac{4}{16}=\frac{11}{16}$$

Q.9. Three faces of a fair die are yellow, two faces red and one blue. The die is tossed three times. The probability that the colours, yellow, red and blue, appear in the first, second and the third tosses respectively is ....(1992 - 2 Marks)

Ans. Sol. Sample space = {Y, Y,Y, R, R, B} where Y stands for yellow colour, R for red and B for blue.

Prob. that the colours yellow, red and blue appear in the first second, and third tosses respectively $\frac{3}{6}\times\frac{2}{6}\times\frac{1}{6}=\frac{1}{36}$

Q.10. If two events A and B are such that P(A^c) = 0.3, P(B) = 0.4 and P (A ∩B ^{c}) = 0.5, then P(B/(A ∪ B^c)] = ... (1994 - 2 Marks)

Ans. Sol. Given that P (A^c) = 0.3, P (B) = 0.4 and P (A ∩ B^c) = 0.5 then P [B/ (A ∩ B^c)]

$$= \frac{P[B\cap(A\cup B^c)]}{P(A\cup B^c)}$$

$$= \frac{P((B\cap A)\cup(B\cap B^c))}{P(A\cup B^c)}$$

$$= \frac{P(A\cap B)}{P(A)+P(B^c)-P(A\cap B^c)}$$

$$= \frac{P(A)-P(A\cap B^c)}{1-P(A^c)+1-P(B)-P(A\cap B^c)}$$

$$= \frac{1-0.3-0.5}{1-0.3+1-0.4-0.5}=\frac{0.2}{0.8}=\frac{1}{4}$$

True False of Probability, Past year Questions JEE Advance

Q. 1. If the letters of the word "Assassin" are written down at random in a row, the probability that no two S's occur together is 1/35 (1983 - 1 Mark)

Ans. Sol. Let E be the event "No two S's occur together".

A, A, I, N can be arranged in $\frac{4!}{2!} = 12$ ways

– A – A – I – N – Creating 5 places for 4 S. Out of 5 places 4 can be selected in ${}^5C_4 = 5$ ways.

No two S's occur together in = 12 × 5 = 60 ways Total no.of arranging all letters of word 'assassin'

$= \frac{8!}{4!2!} = 840$

Req. prob. $= \frac{60}{840} = \frac{1}{14}$ \ Statement is False.

Q.2. If the probability for A to fail in an examination is 0.2 and that for B is 0.3, then the probability that either A or B fails is 0.5. (1989 - 1 Mark)

Ans. Sol. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A) P(B)$

[∵A and B are independent events] $= 0.2 + 0.3 - 0.2 \times 0.3 = 0.5 - 0.06 = 0.44 \neq 0.5$

The statement is false.

Subjective questions of Probability, Part -1

Q.1. Balls are drawn one-by-one without replacement from a box containing 2 black, 4 white and 3 red balls till all the balls are drawn. Find the probability that the balls drawn are in the order 2 black, 4 white and 3 red. (1978)

Ans. Sol. To draw 2 black, 4 white and 3 red balls in order is same as arranging two black balls at first 2 places, 4 white at next 4 places, (3^{rd} to 6^{th} place) and 3 red at still next 3 places (7^{th} to 9^{th} place), i.e., $B_1B_2\ W_1W_2W_3W_4R_1\ R_2\ R_3$, which can be done in 2! × 4! × 3! ways. And total ways of arranging all 2 + 4 + 3 = 9 balls is 9!

∴ Required probability = $\frac{2!\times4!\times3!}{9!}=\frac{1}{1260}$

Q.2. Six boys and six gir ls sit in a row randomly. Find the probability that (i) the six girls sit together (ii) the boys and girls sit alternately. (1979)

Ans. Sol. (i) 6 boys and 6 girls sit in a row randomly.

Total ways of their seating = 12!

No. of ways in which all the 6 girls sit together = 6! × 7! (considering all 6 girls as one person)

∴ Probability of all girls sitting together

$$=\frac{6!\times7!}{12!}=\frac{720}{12\times11\times10\times9\times8}=\frac{1}{132}$$

(ii) Staring with boy, boys can sit in 6! ways leaving one place between every two boys and one a last.

B _ B _ B _ B _ B_ B_

These left over places can be occupied by girls in 6! ways.

∴ If we start with boys. no. of ways of seating boys and girls alternately = 6!× 6!

In the similar manner, if we start with girl, no. of ways of seating boys and girls alternately = 6! × 6!

G _ G _ G _ G _ G_ G _

Thus total ways of alternate seating arrangements = 6! × 6! + 6! × 6!

= 2 × 6! × 6!

∴ Probability of making alternate seating arrangement for 6 boys and 6 girls

$$= \frac{2\times 6!\times 6!}{12!} = \frac{2\times 720}{12\times 11\times 10\times 9\times 8\times 7} = \frac{1}{462}$$

Q.3. An anti -aircraft gun can take a maximum of four shots at an enemy plane moving away from it.The probabilities of hitting the plane at the first, second, third and fourth shot are 0.4, 0.3, 0.2 and 0.1 respectively. What is the probability that the gun hits the plane ? (1981 - 2 Marks)

Ans. Sol. (a) Let us define the events as :

E_1 ≡ First shot hits the target plane,

E_2 ≡ Second shot hits the target plane

E_3 ≡ third shot hits the target plane,

E_4 ≡ fourth shot hits the target plane then

ATQ, P (E_1) = 0.4; P (E_2) = 0.3; P (E_3) = 0.2; P (E_4) = 0.1

$$\Rightarrow \quad P(\bar{E}_1) = 1 - 0.4 = 0.6;\ P(\bar{E}_2) = 1 - 0.3 = 0.7$$

$$P(\bar{E}_3) = 1 - 0.2 = 0.8;\ P(\bar{E}_4) = 1 - 0.1 = 0.9$$

(where $\bar{E}_1$ ' denotes not happening of E_1)Now the gun hits the plane if at least one of the four shots hit the plane.

Also, P (at least one shot hits the plane).

= 1– P (none of the shots hits the plane)

$= 1 - P(\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_3 \cap \bar{E}_4)$

$= 1 - P(\bar{E}_1).P(\bar{E}_2).P(\bar{E}_3).P(\bar{E}_4)$

[Using multiplication them for independent events] = 1– 0.6 × 0.7 × 0.8 × 0.9 = 1– 0.3024 = 0.6976

Q.4. A and B are two candidates seeking admission in IIT. The probability that A is selected is 0.5 and the probability that both A and B are selected is atmost 0.3. Is it possible that the probability of B getting selected is 0.9? (1982 - 2 Marks)

Ans. Sol. Let A denote the event that the candidate A is selected and B the event that B is selected. It is given that P (A) = 0.5 … (1)

P (A ∩ B) ≤ 0.3 … (2)

Now, P (A) + P (B) – P (A ∩ B) = P (A ∪ B) ≤ 1 or 0.5 + P (B) – P (A ∩ B) < 1 [Using (1)]

or P (B) ≤ 0.5 + P (A ∩ B) ≤ 0.5 + 0.3 [Using (2)] or P (B) ≤ 0.8

∴P (B) cannot be 0.9.

Q.5. Cards are drawn one by one at random from a well -shuffled full pack of 52 playing cards until 2 aces are obtained for the first time. If N is the number of cards required to be drawn, then show that $P_r\{N = n\} = \frac{(n-1)(52-n)(51-n)}{50 \times 49 \times 17 \times 13}$ **where 2 ≤ n ≤ 50 (1983 - 3 Marks)**

Ans. Sol. We must have one ace in (n – 1) attempts and one ace in the nth attempt. The probability of drawing one ace in first (n – 1) attempts is and other one ace in the nth attempt is, $\frac{{}^4C_1 \times {}^{48}C_{n-2}}{{}^{52}C_{n-1}}$ and other one ace in the nth attempt is,

$\frac{{}^3C_1}{[52-(n-1)]} = \frac{3}{53-n}$

Hence the required probability,

$$= \frac{4.48!}{(n-2)!(50-n)!} \times \frac{(n-1)!(53-n)}{52!} \times \frac{3}{53-n}$$

$$= \frac{(n-1)(52-n)(51-n)}{50.49.17.13}$$

Q.6. A, B, C are events such that (1983 - 2 Marks) P (A) = 0.3, P(B) = 0.4, P (C) = 0.8

P (AB) = 0.08, P (AC) = 0.28; P (ABC)=0.09
If P(A ∪ B ∪ C)≥ 0.75 , then show that P (BC) lies in the interval 0.23 ≤ x ≤ 0.48

Ans. Sol. Given that P (A) = 0.3, P(B) = 0.4, P (C) = 0.8

P (AB) = 0.08, P (AC) = 0.28, P(ABC) = 0.09

P (A ∪ B ∪ C) ≥ 0.75

To find P (BC) = x (say) Now we know, P (A ∪ B ∪ C) = P (A) + P (B) + P (C)

– P (AB) – P (BC) – P (CA) + P (ABC)

⇒ P (A ∪ B ∪ C) = 0.3 + 0.4 + 0.8

– 0.08 – x – 0.28 + 0.09 = 1.23 – x

Also we have, P (A ∪ B ∪ C) ≥ 0.75 and P (A ∪ B ∪ C) ≤ 1

∴ 0.75 ≤ P (A ∪ B ∪ C) ≤ 1

⇒ 0.75 ≤ 1.23 – x ≤ 1 ⇒ 0.23 ≤ x ≤ 0.48

Q.7. In a certain city only two newspapers A and B are published, it is known that 25% of the city population reads A and 20% reads B while 8% reads both A and B. It is also known that 30% of those who read A but not B look into advertisements and 40% of those who read B but not A look into advertisements while 50% of those who read both A and B look into advertisements. What is the percentage of the population that reads an advertisement? (1984 - 4 Marks)

Ans. Sol. Let P (A) denotes the prob. of people reading newspaper A and P (B) that of people reading newspaper B

Then, $P(A)=\frac{25}{100}=0.25$

$P(B)=\frac{20}{100}=0.20, \quad P(AB)=\frac{8}{100}=0.08$

Prob. of people reading the newspaper A but not B = P (AB^c) = P (A) – P (AB) = 0.25 – 0.08 = 0.17

Similarly, P $(A^c B)$ = P (B) – P (AB) = 0.20 – 0.08 = 0.12

Let E be the event that a person reads an advertisement.

Therefore, ATQ, P (E / AB^c) $=\frac{30}{100}; P(E/A^cB)=\frac{40}{100}$

$P(E/AB)=\frac{50}{100}$

∴ By total prob. theorem (as AB^c, A^cB and AB are mutually exclusive)

P (E) = P (E / AB^c) P (ABc) + P(E / A^cB) P (A^cB) + P (E / AB) . P (AB)

$=\frac{30}{100}\times 0.17+\frac{40}{100}\times 0.12+\frac{50}{100}\times 0.08$

= 0.051 + 0.048 + 0.04 = 0.139.

Thus the population that reads an advertisement is 13.9%.

Q.8. In a multiple-choice question there are four alternative answers, of which one or more are correct. A candidate will get marks in the question only if he ticks the correct answers. The candidate decides to tick the answers at random, if he is allowed upto three chances to answer the questions, find the probability that he will get marks in the question s. (1985 - 5 Marks)

Ans. Sol. The total number of ways of ticking the answers in any one attempt = 24 – 1 = 15.

The student is taking chance at ticking the correct answer, It is reasonable to assume that in order to derive maximum benefit, the three solutions which he submit must be all different.

∴ n = total no. of ways = ${}^{15}C_3$ m = the no. of ways in which the correct solution is excluded = ${}^{14}C_3$

Hence the required probability $= 1 - \frac{{}^{14}C_3}{{}^{15}C_3} = 1 - \frac{4}{5} = \frac{1}{5}$

Q.9. A lot contain s 20 articles. The probability that the lot contains exactly 2 defective articles is 0.4 and the probability that the lot contains exactly 3 defective articles is 0.6. Articles are drawn from the lot at random one by one without replacement and are tested till all defective articles are found.
What is the probability that the testing procedure ends at the twelth testing. (1986 - 5 Marks)

Ans. Sol. Let A_1 be the event that the lot contains 2 defective articles and A_2 the event that the lot contains 3 defective articles.

Also let A be the event that the testing procedure ends at the twelth testing. Then according to the question :

$P(A_1) = 0.4$ and $P(A_2) = 0.6$ Since $0 < P(A_1) < 1$, $0 < P(A_2) < 1$, and $P(A_1) + P(A_2) = 1$

∴ The events A_1, A_2 form a partition of the sample space.

Hence by the theorem of total probability for compound events, we have

$P(A) = P(A_1)\, P(A/A_1) + P(A_2)\, P(A/A_2)$ … (1)

Here $P(A/A_1)$ is the probability of the event the testing procedure ends at the twelth testing when the lot contains 2 defective articles.

This is possible when out of 20 articles, first 11 draws must contain 10 non defective and 1defective article and 12th draw must give a defective article.

$$\therefore \quad P(A/A_1) = \frac{{}^{18}C_{10} \times {}^{2}C_1}{{}^{20}C_{11}} \times \frac{1}{9} = \frac{11}{190}$$

Similarly, $P(A/A_2) = \frac{{}^{17}C_9 \times {}^{3}C_1}{{}^{20}C_{11}} \times \frac{1}{9} = \frac{11}{228}$

Now substituting the values of P (A /A_1) and P (A /A_2) in eq. (1), we get

$$P(A) = 0.4 \times \frac{11}{190} + 0.6 \times \frac{11}{228} = \frac{11}{475} + \frac{11}{380} = \frac{99}{1900}$$

Q.10. A man takes a step forward with probability 0.4 an d backwards with probability 0.6 Find the probability that at the end of eleven steps he is one step away from the starting point. (1987 - 3 Marks)

Ans. Sol. Since the man is one step away from starting point means that either (i) man has taken 6 steps forward and 5 steps backward. or (ii) man has taken 5 steps forward and 6 steps backward.

Taking movement 1 step forward as success and 1 step backward as failure.

∴ p = Probability of success = 0.4 and q = Probability of failure = 0.6

∴ Required probability = P (X = 6 or X = 5) = P (X = 6) + P (X = 5)

$= {}^{11}C_6\, p^6q^5 + {}^{11}C_5\, p^5\, q^6 = {}^{11}C_5\, (p^6q^5 + p^5\, q^6) = {}^{11}C_5\, (p + q)\, (p^5 q^5)$

$$= \frac{11.10.9.8.7}{1.2.3.4.5}(0.4+0.6)(0.4\times 0.6)^5$$

$= 462 \times 1 \times (0.24)^5 = 0.37$

Hence the required prob. = 0.37

Q.11. A box contains 2 fifty paise coins, 5 twenty five paise coins and a certain fixed number N (≥ 2) of ten and five paise coins. Five coins are taken out of the box at random. Find the probability that the total value of these 5 coins is less than one rupee and fifty paise. (1988 - 3 Marks)

Ans. Sol. Here the total number of coins is N + 7.Therefore the total number of ways of choosing 5 coins out of N + 7 is ${}^{N+7}C_5$.

Let E denotes the event that the sum of the values of the coins is less than one rupee and fifty paise.

Then E' denotes the event that the total value of the five coins is equal to or more than one rupee and fifty paise.

The number of cases favourable to E' is

$= {}^2C_1 \times {}^5C_4 \times {}^NC_0 + {}^2C_2 \times {}^5C_3 \times {}^NC_0 + {}^2C_2 \times {}^5C_2 \times {}^NC_1$

$= 2 \times 5 + 10 + 10N = 10\,(N + 2)$

$$\therefore\ P(E') = \frac{10(N+2)}{{}^{N+7}C_5} \Rightarrow P(E) = 1 - P(E') = 1 - \frac{10(N+2)}{{}^{N+7}C_5}$$

Q.12. Suppose the probability for A to win a game against B is 0.4. If A has an option of playing either a "best of 3 games" or a "best of 5 games" match against B, which option should be choose so that the probability of his winning the match is higher? (No game ends in a draw). (1989 - 5 Marks)

Ans. Sol. The probability p_1 (say) of winning the best of three games is = the prob. of winning two games + the prob. of winning three games.

$= {}^3C_2\,(0.6)\,(0.4)^2 + {}^3C_3\,(0.4)^3$ [Using Binomial distribution]

Similarly the probability of winning the best five games is p_2 (say) = the prob. of winning three games + the prob. of winning four games + the prob. of winning 5 games

$= {}^5C_3(0.6)^2(0.4)^3 + {}^5C_4(0.6)(0.4)^4 + {}^5C_5(0.4)^5$

We have $p_1 = 0.288 + 0.064 = 0.352$ and $p_2 = 0.2304 + 0.0768 + 0.01024 = 0.31744$ As $p_1 > p_2$

$\therefore$ A must choose the first offer i.e. best of three games.

Q.13. A is a set containing n elements. A subset P of A is chosen at random. The set A is reconstructed by replacing the elements of P. A subset Q of A is again chosen at random.

Find the probability that P and Q have no common elements. (1990 - 5 Marks)

Ans. Sol. Let $A = \{a_1, a_2, a_3, \ldots, a_n\}$

For each a_i, $1 \le i \le n$, there aries 4 cases

(i) $a_i \in P$ and $a_i \in Q$ (ii) $a_i \notin P$ and $a_i \in Q$ (iii) $a_i \in P$ and $a_i \notin Q$ (iv) $a_i \notin P$ and $a_i \notin Q$

∴ Total no. of ways of choosing P and Q is 4^n. Here case (i) is not favourable as P ∩ Q = f

∴ For each element there are 3 favourable cases and hence total no. of favourable cases = 3^n.

Hence prob. $(P \cap Q = \phi) = \frac{3^n}{4^n} = \left(\frac{3}{4}\right)^n$.

Q.14. In a test an examine either guesses or copies or knows the answer to a multiple choice question with four choices. The probability that he make a guess is 1/3 and the probability that he copies the answer is 1/6. The probability that his answer is correct given that he copied it, is 1/8. Find the probability that he knew the answer to the question given that he correctly answered it. (1991 - 4 Marks)

Ans. Sol. Let us define the events :

A_1 ≡ the examinee guesses the answer,,

A_2 ≡ the examinee copies the answer

A_3 ≡ the examinee knows the answer,,

A ≡ the examinee answers correctly.

Then, $P(A_1) = \frac{1}{3}; P(A_2) = \frac{1}{6}$

As any one happens out of A_1, A_2, A_3, these are mutually exclusive and exhaustive events.

∴ P (A_1) + P (A_2) + P (A_3) = 1

$$P(A_3) = 1 - \frac{1}{3} - \frac{1}{6} = \frac{6-2-1}{6} = \frac{3}{6} = \frac{1}{2}$$

Also we have, P (A/A_1) = $\frac{1}{4}$

[∵ out of 4 choices only one is correct.] $| P(A/A_2) = \frac{1}{8}$

(given) P (A/A_3) = 1

[If examinee knows the ans., it is correct. i.e. true event]

To find P (A_3/A). By Baye's them, P (A_3/A)

$$=\frac{P(A/A_3)P(A_3)}{P(A/A_1)P(A_1)+P(A/A_2)P(A_2)+P(A/A_3)P(A_3)}$$

$$=\frac{1.\frac{1}{2}}{\frac{1}{4}.\frac{1}{3}+\frac{1}{8}.\frac{1}{6}+1.\frac{1}{2}} \quad =\frac{1/2}{\frac{29}{48}}=\frac{1}{2}\times\frac{48}{29}=\frac{24}{29}$$

Q.15. A lot contains 50 defective and 50 non defective bulbs. Two bulbs are drawn at random, one at a time, with replacement.
The events A, B, C are defined as (1992 - 6 Marks)
A = (the first bulb is defective)
B = (the second bulb is non-defective)
C = (the two bulbs are both defective or both non defective) Determine whether
(i) A, B, C are pairwise independent (ii) A, B, C are independent

Ans. Sol. Let X = defective and Y = non defective. Then all possible outcomes are {XX, XY, YX, YY}

Also $P(XX)=\frac{50}{100}\times\frac{50}{100}=\frac{1}{4}$,

$=\frac{50}{100}\times\frac{50}{100}=\frac{1}{4}\quad P(YX)=\frac{50}{100}\times\frac{50}{100}=\frac{1}{4}$,

$P(YY)=\frac{50}{100}\times\frac{50}{100}=\frac{1}{4}$

Here, A = XX ∪ XY ; B = XY ∪ YY ; C = XX ∪ YY

∴ P (A) = P(XX) + P (XY) $=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$

∴ P (B) = P (XY) + P (YX) $=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$

P (C) = P (XX) + P (YY) = $\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$

Now, $P(AB)=P(XY)=\frac{1}{4}=P(A).P(B)$

∴ A and B are independent events.

$$P(BC) = P(YX) = \frac{1}{4} = P(B).P(C)$$

∴ B and C are independent events.

$$P(CA) = P(XX) = \frac{1}{4} = P(C).P(A)$$

∴ C and A are independent events.

P (ABC) = 0 (impossible event) $\neq$ P (A) P (B) P (C)

∴ A, B, C are dependent events.

Thus we can conclude that A, B, C are pairwise independent but A, B, C are dependent events.

Subjective questions of Probability, Part -2

Q.16. Numbers are selected at random, one at a time, from the twodigit numbers 00, 01, 02....., 99 with replacement. An event E occurs if only if the product of the two digits of a selected number is 18. If four numbers are selected, find probability that the event E occurs at least 3 times. (1993 - 5 Marks)

Ans. Sol. The given numbers are 00, 01, 02, …, 99.

These are total 100 numbers, out of which the numbers, the product of whose digits is 18, are 29, 36, 63 and 92.

$$\therefore \quad p = P(E) = \frac{4}{100} = \frac{1}{25} \Rightarrow q = 1 - p = \frac{24}{25}$$

From Binomial distribution P (E occurring at least 3 times) = P (E occurring 3 times) + P (E occurring 4 times)

$$^4C_3 p^3 q + {}^4C_4 p^4 = 4 \times \left(\frac{1}{25}\right)^3 \left(\frac{24}{25}\right) + \left(\frac{1}{25}\right)^4 = \frac{97}{(25)^4}$$

Q.17. An unbiased coin is tossed. If the result is a head, a pair of unbiased dice is rolled and the number obtained by adding the numbers on the two faces is noted. If the result is a tail, a card from a well shuffled pack of eleven cards numbered 2, 3, 4,.....12 is picked and the number on the card is noted.

What is the probability that the noted number is either 7 or 8? (1994 - 5 Marks)

Ans. Sol. E_1 ≡ number noted is 7, E_2 ≡ number notes is 8, H ≡ getting head on coin, T ≡ getting tail on coin.

Then by total probability theorem, P (E_1) = P (H) P (E_1/H) + P (T) P (E_1/T)

and P (E_2) = P (H) P (E_2/H) + P (T) P (E_2/T)

Where $P(H) = \frac{1}{2}; P(T) = \frac{1}{2}$

P (E_1/H) = prob. of getting a sum of 7 on two dice. Here favourable cases are {(1, 6) , (6, 1), (2, 5), (5, 2), (3, 4), (4, 3)}

$\therefore\ P(E_1/H)=\frac{6}{36}=\frac{1}{6}$

Also P (E_1/ T) = prob. of getting '7' numbered card out of 11 cards $=\frac{1}{11}$

P (E_2 / H) = Prob. of getting a sum of 8 on two dice. Here favourable cases are {(2, 6) (6, 2) (4, 4), (5, 3), (3, 5)}

$\therefore$ P (E_2/H) = $\frac{5}{36}$.

P (E_2 / T) = prob. of getting '8' numbered card out of 11 car ds $\frac{1}{11}$

$$\therefore\ P(E_1)=\frac{1}{2}\times\frac{1}{6}+\frac{1}{2}\times\frac{1}{11}=\frac{1}{12}+\frac{1}{22}=\frac{11+6}{132}=\frac{17}{132}$$

$$P(E_2)=\frac{1}{2}\times\frac{5}{36}+\frac{1}{2}\times\frac{1}{11}=\frac{1}{2}\left[\frac{55+36}{396}\right]=\frac{91}{792}$$

Now E_1 and E_2 are mutually exclusive events therefore

P (E_1 or E_2) = P (E_1) + P (E_2) $=\frac{17}{132}+\frac{91}{792}$

$=\frac{102+91}{792}=\frac{193}{792}=0.2436.$

Q.18. In how many ways three girls and nine boys can be seated in two vans, each having numbered seats, 3 in the front and 4 at the back? How many seating arrangements are possible if 3 girls should sit together in a back row on adjacent seats?

Now, if all the seating arrangements are equally likely, what is the probability of 3 girls sitting together in a back row on adjacent seats? (1996 - 5 Marks)

Ans. Sol. We have 14 seats in two vans. And there are 9 boys and 3 girls.

The no. of ways of arranging 12 people on 14 seats without restriction is $^{14}P_{12}=\frac{14!}{2!}=7(13!)$

Now the no. of ways of choosing back seats is 2. And the no. of ways of arranging 3 girls on adjacent seats is 2 (3!).

And the no. of ways of arranging 9 boys on the remaining 11 seats is ${}^{11}P_9$
Therefore, the required number of ways

$$= 2.(2.3!).\, {}^{11}P_9 = \frac{4.3!.11!}{2!} = 12!$$

Hence, the probability of the required event $= \frac{12!}{7.13!} = \frac{1}{91}$

Q.19. If p and q are chosen randomly from the set {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, with replacement, determine the probability that the roots of the equation $x^2 + px + q = 0$ are real. (1997 - 5 Marks)

Ans. Sol.

The required probability = 1– (probability of the event that the roots of $x^2 + px + q = 0$ are non-real if and only if $p^2 – 4q < 0$ i.e. if $p^2 < 4q$.

We enumerate the possible values of p and q, for which this can happen in the following table.

q	p	Number of pairs of p,q
1	1	1
2	1, 2	2
3	1, 2, 3	3
4	1, 2, 3	3
5	1, 2, 3, 4	4
6	1, 2, 3, 4	4
7	1, 2, 3, 4, 5	5
8	1, 2, 3, 4, 5	5
9	1, 2, 3, 4, 5	5
10	1, 2, 3, 4, 5, 6	6

Thus, the number of possible pairs = 38. Also, the total number of possible pairs is 10 × 10 = 100.

∴ The required probability = $1 - \frac{38}{100} = 1 – 0.38 = 0.62$.

Q.20. Three players, A, B and C, toss a coin cyclically in that order (that is A, B, C, A, B, C,A,B, ...) till a head shows. Let p be the probability that the coin shows a head. Let a, b and g be, respectively, the probabilities that A, B and C gets the first head. Prove that b = (1 - p) a. Determine a, b and g (in terms of p). (1998 - 8 Marks)

Ans. Sol. Given that p is the prob. that coin shows a head then 1– p will be the prob. that coin shows a tail.

Now, α = P (A gets the 1st head in 1st try) + P (A gets the 1st head in 2nd try) + …

$\Rightarrow \alpha = P(H) + P(T)\,P(T)\,P(T)\,P(H) + P(T)\,P(T)\,P(T)\,P(T)\,P(T)\,P(T)\,P(H)$

$= p + (1-p)^3 p + (1-p)^6 p + \ldots\ldots = p\,[1 + (1-p)^3\,(1-p)^6 + \ldots\ldots] = \dfrac{p}{1-(1-p)^3}$

Similarly β = P (B gets the 1st head in 1st try) + P (B gets the 1st head in 2nd try) + ……

$= P(T)\,P(H) + P(T)\,P(T)\,P(T)\,P(T)\,P(H) + \ldots\ldots$

$= (1-p)\,p + (1-p)4\,p + \ldots\ldots \; = \dfrac{(1-p)\,p}{1-(1-p)^3}$ …(ii)

From (i) and (ii) we get b = (1– p) a

Also (i) and (ii) give expresion for α and β in terms of p.

Also $\alpha + \beta + \gamma = 1$ (exhaustive events and mutually exclusive events)

$\Rightarrow \gamma = 1 - \alpha - \beta = 1 - \alpha - (1-p)\alpha$

$$= 1-(2-p)\,\alpha = 1-(2-p)\,\frac{p}{1-(1-p)^3}$$

$$= \frac{1-(1-p)^3-(2p-p^2)}{1-(1-p)^3}$$

$$= \frac{1-1+p^3+3p(1-p)-2p+p^2}{1-(1-p)^3}$$

$$= \frac{p^3-2p^2+p}{1-(1-p)^3} = \frac{p(p^2-2p+1)}{1-(1-p)^3} = \frac{p(1-p)^2}{1-(1-p)^3}$$

Q.21. Eight players P_1, P_2,........P_8 play a knock-out tournament.
It is known that whenever the players P_i and P_j play, the player P_i will win if i < j.
Assuming that the players are paired at random in each round, what is the probability that the player P_4 reaches the final? (1999 - 10 Marks)

Ans. Sol. The number of ways in which $P_1, P_2, \ldots, P_8$ can be paired in four pairs

$$= \frac{1}{4!} \times {}^8C_2 \times {}^6C_2 \times {}^4C_2 \times {}^2C_2 = 105$$

Now, at least two players certainly reach the second round in between P_1, P_2 and P_3. And P_4 can reach in final if exactly two players play against each other in between P_1, P_2, P_3 and remaining player will play against one of the players from P_5, P_6, P_7, P_8 and P_4 plays against one of the remaining three from P_5, P_6, P_7, P_8.

This can be possible in ${}^3C_2 \times {}^4C_1 \times {}^3C_1 = 36$ ways

∴ Prob. that P4 and exactly one of P5…P8 reach second

round $= \frac{36}{105} = \frac{12}{35}$

If P_1, P_i, P_4 and P_j where i = 2 or 3 and j = 5 or 6 or 7 reach the second round then they can be paired in 2 pairs in

$\frac{1}{2!} \times {}^4C_2 \times {}^2C_2$ ways

But P_4 will reach the final if P_1 plays against P_i and P_4 plays against P_j.

Hence the prob. that P_4 reach the final round from the second $= \frac{1}{3}$

∴ prob. that P_4 reach the final is $\frac{12}{35} \times \frac{1}{3} = \frac{4}{35}$

22. A coin has probability p of showing head when tossed. It is tossed n times. Let p_n denote the probability that no two (or more) consecutive heads occur. Prove that $p_1=1$, $p_2=1-p^2$ and $p_n=(1-p).\ p_{n-1} + p(1-p)\ p_{n-2}$ for all $n \geq 3$. (2000 - 5 Marks)

Ans. Sol. Given that the probability of showing head by a coin when tossed = p

∴ Prob. of coin showing a tail = 1– p Now p_n = prob. that no two or more consecutive heads occur when tossed n times.

∴ p_1 = prob. of getting one or more or no head = prob. of H or T = 1

Also p_2 = prob. of getting one H or no H = P (HT) + P (TH) + P (TT)

$= p(1-p) + p(1-p)p + (1-p)(1-p) = 1-p^2$, For $n \geq 3$ p_n = prob. that no two or more consecutive heads occur when tossed n times. = P (last outcome is T) P (no two or more consecutive heads in (n – 1) throw) + P (last outcome is H) P ((n – 1)th throw results in a T) P (no two or more consecutive heads in (n – 2) n throws) = $(1-p)P_{n-1} + p(1-p)p_{n-2}$

Hence Proved.

23. An urn contains m white and n black balls. A ball is drawn at random and is put back into the urn along with k additional balls of the same colour as that of the ball drawn. A ball is again drawn at random. What is the probability that the ball drawn now is white? (2001 - 5 Marks)

Ans. Sol. Let W_1 (B_1) be the event that a white (a back) ball is drawn in the first draw

and let W be the event that a white ball is drawn in the second draw.

Then $P(W) = P(B_1) . P(W/B_1) + P(W_1). P(W/W_1)$

$$= \frac{n}{m+n} \cdot \frac{m}{m+n+k} + \frac{m}{m+n} \cdot \frac{m+k}{m+n+k}$$

$$= \frac{m(n+m+k)}{(m+n)(m+n+k)} = \frac{m}{m+n}$$

24. An unbiased die, with faces numbered 1, 2, 3, 4, 5, 6, is thrown n times and the list of n numbers showing up is noted. What is the probability that, among the numbers 1, 2, 3, 4, 5, 6, only three numbers appear in this list? (2001 - 5 Marks)

Ans. Sol. The total no. of outcomes = 6^n

We can choose three numbers out of 6 in $^6C_3$ ways. By using three numbers out of 6 we can get 3^nsequences of length n.

But these include sequences of length n which use exactly two numbers and exactly one number.

The number of n-sequences which use exactly two numbers = $^3C_2\,[2^n - 1^n - 1^n] = 3(2^n - 2)$ and the number of n sequences which are exactly one number = $(^3C_1)(1^n) = 3$. Thus , the number of sequences, which use exactly three numbers = $^6C_3\,[3^n - 3(2^n - 2) - 3] = {}^6C_3[3^n - 3(2^n) + 3]$

∴ Probability of the required event, $= \frac{{}^6C_3[3^n - 3(2^n) + 3]}{6^n}$

25. A box contains N coins, m of which are fair and the rest are biased. The probability of getting a head when a fair coin is tossed is 1/2, while it is 2/3 when a biased coin is tossed. A coin is drawn from the box at random and is tossed twice. The first time it shows head and the second time it shows tail. What is the probability that the coin drawn is fair? (2002 - 5 Marks)

Ans. Sol. Let E_1 be the event that the coin drawn is fair and E_2 be the event that the coin drawn is biased.

$$\therefore \quad P(E_1) = \frac{m}{N} \text{ and } P(E_2) = \frac{N-m}{N}$$

A is the event that on tossing the coin the head appears first and then appears tail.

∴ $P(A) = P(E_1 \cap A) + P(E_2 \cap A)$

$= P(E_1)\, P(A/E_1) + P(E_2)\, P(A/E_2)$

$$= \frac{m}{N}\left(\frac{1}{2}\right)^2 + \left(\frac{N-m}{N}\right)\left(\frac{2}{3}\right)\left(\frac{1}{3}\right) \quad \dots (1)$$

We have to find the probability that A has happened because of E_1

$$\therefore \quad P(E_1/A) = \frac{P(E_1 \cap A)}{P(A)}$$

$$= \frac{\frac{m}{N}\left(\frac{1}{2}\right)^2}{\frac{m}{N}\left(\frac{1}{2}\right)^2 + \frac{N-m}{N}\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)} \quad \text{(by (1))}$$

$$= \frac{m/4}{m/4 + \frac{2(N-m)}{9}} = \frac{9m}{m+8N}$$

26. For a student to qualify, he must pass at least two out of three exams. The probability that he will pass the 1st exam is p. If he fails in one of the exams then the probability of his passing in the next exam is $\frac{p}{2}$ otherwise it remains the same. Find the probability that he will qualify. (2003 - 2 Marks)

Ans. Sol. Let us consider $E_1 \equiv$ event of passing I exam.

$E_2 \equiv$ event of passing II exam.

$E_3 \equiv$ event of passing III exam.

Then a student can qualify in anyone of following ways

1. He passes first and second exam.

2. He passes first, fails in second but passes third exam.

3. He fails in first, passes second and third exam.

∴ Required probability = $P(E_1) P(E_2/E_1) + P(E_1) P(E_2/E_1) P(E_3/E_2) + P(E_1)$

$P(E_2/E_1) P(E_3/E_2)$

[as an event is dependent on previous one]

$$= p.p + p.(1-p).\frac{p}{2} + (1-p).\frac{p}{2}.p$$

$$= p^2 + \frac{p^2}{2} - \frac{p^3}{2} + \frac{p^2}{2} - \frac{p^3}{2} = 2p^2 - p^3$$

27. A is targeting to B, B and C are targeting to A. Probability of hitting the target by A, B and C are $\frac{2}{3}, \frac{1}{2}$ and $\frac{1}{3}$ respectively..

If A is hit then find the probability that B hits the target and C does not. (2003 - 2 Marks)

Ans. Sol. Let us consider the events

$E_1 \equiv$ A hits B Then $P(E_1) = 2/3$

$E_2 \equiv$ B hits A $P(E_2) = 1/2$

$E_3 \equiv$ C hits A $P(E_3) = 1/3$ $E \equiv$ A is hit

$$P(E) = P(E_2 \cup E_3) = 1 - P(\bar{E}_2 \cap \bar{E}_3)$$

$= 1 - P(\bar{E}_2).P(\bar{E}_3) = 1 - \frac{1}{2}.\frac{2}{3} = \frac{2}{3}$

To find $P(E_2 \cap \bar{E}_3 / E)$

$= \frac{P(E_2 \cap \bar{E}_3)}{P(E)}$ [$\because P(E_2 \cap \bar{E}_3 \cap E) = P(E_2 \cap \bar{E}_3)$ i.e., B hits A and A is hit = B hits A]

$= \frac{P(E_2).P(\bar{E}_3)}{P(E)} = \frac{1/2 \times 2/3}{2/3} = \frac{1}{2}$

28. A an d B ar e two independent events. C is event in wh ich exactly one of A or B occurs. Prove that $P(C) \geq P(A \cup B)P(\bar{A} \cap \bar{B})$ (2004 - 2 Marks)

Ans. Sol. Given that A and B are two independent events. C is the event in which exactly of A or B occurs.

Let P (A) = x, P(B) = y then P(C) = $P(A \cap \bar{B}) + P(\bar{A} \cap B) = P(A)P(\bar{B}) + P(\bar{A})P(B)$

[∵ If A and B are independent so are ‘A and $\bar{B}$’ and $\bar{A}$ and B’.]

⇒ P (C) = x (1– y) + y (1– x) … (1)

Now consider, $P(A \cup B)P(\bar{A} \cap \bar{B})$

$= [P(A) + P(B) - P(A)P(B)]\,[P(\bar{A})P(\bar{B})]$

= (x + y – xy) (1– x) (1– y)

= (x + y) (1– x) (1– y) –xy (1–x) (1–y) ≤ (x + y) (1– x) (1– y) [∵ x,y ∈ (0,1)]

= x (1– x)(1– y) + y (1– x) (1– y)

= x (1– y) + y (1– x) – x^2 (1– y) – y^2 (1– x) ≤ x(1 - y) + y$(1-x)^3$

= P (C) [Using eqn (1)]

Thus $P(C) \geq P(A \cup B)P(\bar{A} \cap \bar{B})$ is proved.

29. A box contains 12 red and 6 white balls. Balls are drawn from the box one at a time without replacement. If in 6 draws there are at least 4 white balls, find the

probability that exactly one white is drawn in the next two draws. (Binomial coefficients can be left as such) (2004 - 4 Marks)

Ans. Sol. Let us define the following events

A ≡ 4 white balls are drawn in first six draws

B ≡ 5 white balls are drawn in first six draws

C ≡ 6 white balls are drawn in first six draws

E ≡ exactly one white ball is drawn in next two draws (i.e. one white and one red)

Then P(E) = P (E/A) P (A) + P(E/B) P (B) + P (E/C) P (C) But P (E/C) = 0 [As there are only 6 white balls in the bag.]

P (E) = P (E/A) P (A) + P(E/B) P(B)

$$= \frac{^{10}C_1 \times {^2C_1}}{^{12}C_2} \cdot \frac{^{12}C_2 \times {^6C_4}}{^{18}C_6} + \frac{^{11}C_1 \times {^1C_1}}{^{12}C_2} \cdot \frac{^{12}C_1 \times {^6C_5}}{^{18}C_6}$$

30. A person goes to office either by car, scooter, bus or train, the probability of which being $\frac{1}{7}, \frac{3}{7}, \frac{2}{7}$ and $\frac{1}{7}$ respectively..

Probability that he reaches office late, if he takes car, scooter, bus or train is $\frac{2}{9}, \frac{1}{9}, \frac{4}{9}$ and $\frac{1}{9}$ respectively. Given that he reached office in time, then what is the

probability that he travelled by a car. (2005 - 2 Marks)

Ans. Sol. Let us define the following events

C ≡ person goes by car,

S ≡ person goes by scooter,

B ≡ person goes by bus,

T ≡ person goes by train,

L ≡ person reaches late

Then we are given in the question

$$P(C)=\frac{1}{7};P(S)=\frac{3}{7};P(B)=\frac{2}{7};P(T)=\frac{1}{7}$$

$$P(L/C)=\frac{2}{9};P(L/S)=\frac{1}{9};\ P(L/B)=\frac{4}{9};P(L/T)=\frac{1}{9}$$

To find the prob. $P(C/\bar{L})$ [∵ reaches in time ≡ not late] Using Baye's theorem

$$P(C/\bar{L})\ =\frac{P(\bar{L}/C)P(C)}{P(\bar{L}/C)P(C)+P(\bar{L}/S)P(S)}\ \ldots(i)$$

$$+P(\bar{L}/B)P(B)+A(\bar{L}/T)P(T)$$

Now, $P(\bar{L}/C)=1-\frac{2}{9}=\frac{7}{9};\ P(\bar{L}/S)=1-\frac{1}{9}=\frac{8}{9}$

$$P(\bar{L}/B)=1-\frac{4}{9}=\frac{5}{9};\ \ P(\bar{L}/T)=1-\frac{1}{9}=\frac{8}{9}$$

Substituting these values in eqn. (i) we get

$$P(C/\bar{L})=\frac{\frac{7}{9}\times\frac{1}{7}}{\frac{7}{9}\times\frac{1}{7}+\frac{8}{9}\times\frac{3}{7}+\frac{5}{9}\times\frac{2}{7}+\frac{8}{9}\times\frac{1}{7}}$$

$$=\frac{7}{7+24+10+8}=\frac{7}{49}=\frac{1}{7}$$

Integar Type ques of Probability, Past year Questions JEE Advance

Q.1. Of the three independent events E_1, E_2 and E_3, the probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. Let the probability p that none of events E_1, E_2or E_3 occurs satisfy the equations (α 2β)p = αβ and (β – 3γ)p = 2bg.

All the given probabilities are assumed to lie in the interval (0, 1). (JEE Adv. 2013)

Then $\frac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3}$

Ans. Sol. (6) Let $P(E_1) = x$; $P(E_2) = y$, $P(E_3) = z$

$P(\text{only } E_1) = x(1 - y)(1 - z) = \boldsymbol{\alpha}$

$P(\text{only } E_2) = (1 - x)\, y\, (1 - z) = \boldsymbol{\beta}$

$P(\text{only } E_3) = (1 - x)(1 - y)\, z = g$

$P(\text{none}) = (1 - x)(1 - y)(1 - z) = p.$

Now given $(\boldsymbol{\alpha} - 2\boldsymbol{\beta})\, p = \boldsymbol{\alpha\beta}$

$\Rightarrow x = 2y$ an d$(\boldsymbol{\beta} - 3r)p = 2\boldsymbol{\beta}r \Rightarrow y = 3z$

$\therefore x = 6z$

Hence

Q.2. The minimum number of times a fair coin needs to be tossed, so that the probability of getting at least two heads is at least 0.96, is (JEE Adv. 2015)

Ans. Sol. $P(x > 2) > 0.96$

$\Rightarrow 1 - P(x = 0) - P(x = 1) > 0.96$

$\Rightarrow P(x = 0) + P(x = 1) < 0.04$

$$\Rightarrow \quad \left(\frac{1}{2}\right)^n + n\left(\frac{1}{2}\right)^n \leq 0.04$$

$$\Rightarrow \quad \frac{n+1}{2^n} \leq \frac{1}{25} \Rightarrow \frac{2^n}{n+1} \geq 25$$

$\Rightarrow$ Minimum value of n is 8.